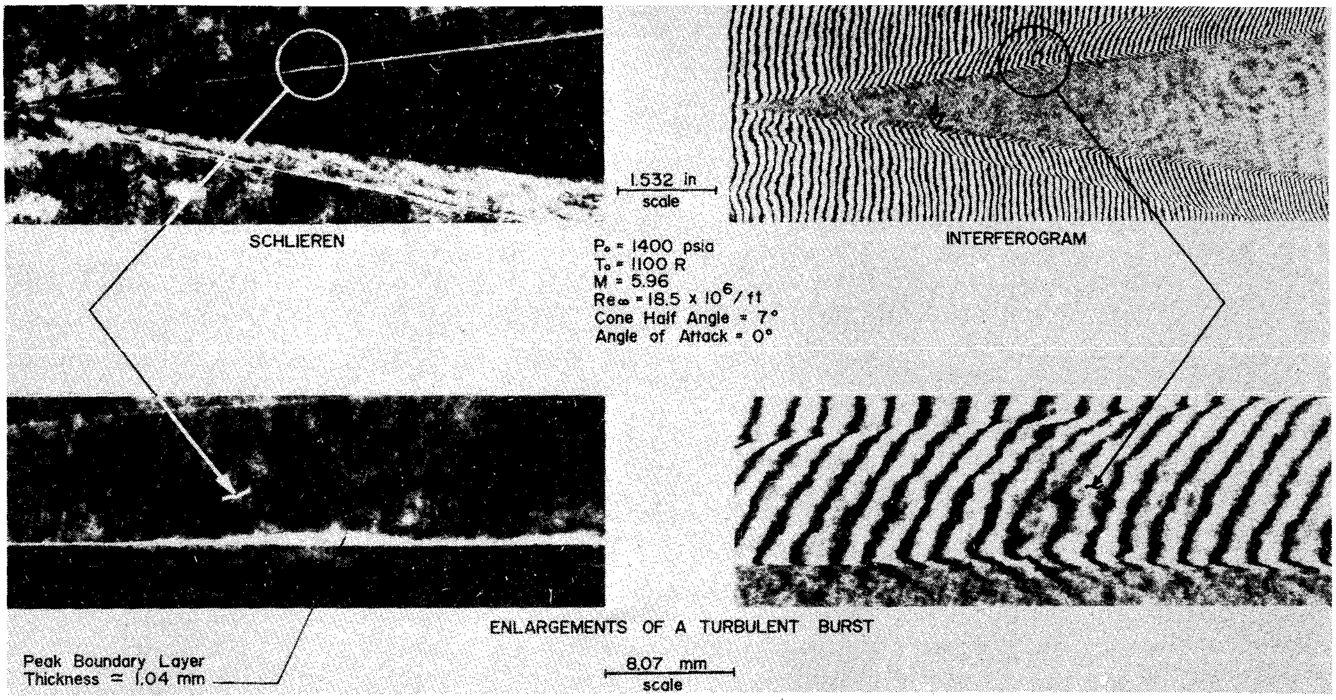

Fig. 1 Holographic Schlieren and interferometric depiction of boundary-layer transition on slender sharp tip cone.

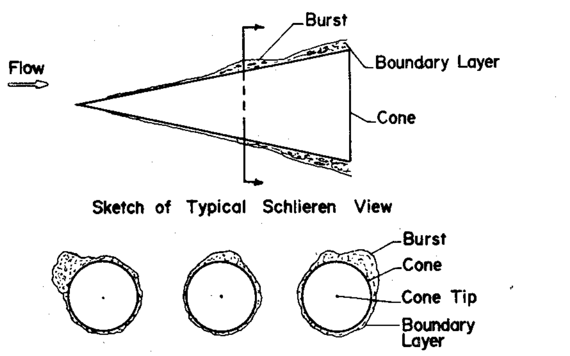

Fig. 2 Illustration of possible orientations of a turbulent burst on a slender sharp tip cone at zero angle of attack. Note the different cross-sectional views of possible burst configurations that could produce the optical effect illustrated above.

shapes could produce the optical results shown in Fig. 1, and some possibilities are illustrated in Fig. 2. Although these cross sections are imaginative, they illustrate that a single view of a burst is inadequate for obtaining a complete description.

A second objective of this holographic application is to obtain density data from the interferometric measurements, but since the geometry and orientation of the bursts are unknown, flow symmetry cannot be assumed, which means the data reduction schemes normally used may be inappropriate. However, the limitations of the data reduction schemes cannot be determined at this time because no quantitative results are available for detailed analysis. Density profile data have not been obtained because precise fringe measurements cannot be made with the existing equipment.

These holographic measurements offer a means to investigate the fundamental structure of boundary-layer turbulence and transition. Acquiring instantaneous measurements of the bursting phenomenon offers the additional possibility to obtain insights on the fluid mechanics resulting in transition. New apparatus are being built to overcome the inadequacies of the existing equipment, and when completed, the new equipment will be used to reinvestigate the cone study shown in Fig. 1, and to boundary layers on other geometries both at zero and small angle of attack.

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Unsteady Aerodynamic Modeling for Arbitrary Motions

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Introduction

UNSTEADY aerodynamic loads due to arbitrary stable motions are of interest in the application of active control techniques to flexible vehicles. These studies are well

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suited to Laplace transform analysis techniques. Unfortunately, unsteady aerodynamic loads have traditionally been derived in a manner that was assumed to be valid only for simple harmonic motions of the airfoil or wing. Edwards[1] and Edwards, Ashley, and Breakwell[2] show that these loads may be evaluated for arbitrary motions. This Note summarizes their results.

The Generalized Theodorsen Function

The solution for the loads due to simple harmonic oscillations of a wing section in incompressible flow was first given by Theodorsen.[3] The circulatory lift on the airfoil is

$$L_c=\rho U\int_b^{x_0}\frac{\xi}{\sqrt{\xi^2-b^2}}\gamma_w(\xi,t)\,d\xi \tag{1}$$

while the Kutta condition of smooth flow off the trailing edge is enforced by requiring

$$\frac{2}{\pi}\int_0^{\pi}\frac{w_a\sin^2\varphi}{\cos\varphi-1}\,d\varphi+\frac{1}{\pi b}\int_b^{b+Ut}\sqrt{\frac{\xi+b}{\xi-b}}\,\gamma_w(\xi,t)\,d\xi=0 \tag{2}$$

In these expressions $\gamma_w(\xi,t)$ is the unknown wake vortex strength distribution and w_a is the known downwash at the airfoil. The first integral in Eq. (2) is the tangential velocity at the trailing edge due to the noncirculatory source-sink flow. The second integral gives the corresponding velocity for the vortex flow. One-half of the first expression in Eq. (2) may be evaluated as $Q(t)=U\alpha+\dot{h}b+b(0.5-a)\dot{\alpha}$. To obtain a solution, Theodorsen assumed that the airfoil motion was simple harmonic and had endured for an infinite time, thereby identifying the integrals as modified Bessel functions; for example

$$K_0(s)=\int_1^{\infty}\frac{e^{-s\lambda}}{\sqrt{\lambda^2-1}}\,d\lambda;\quad Re(s)>0 \tag{3}$$

The circulatory lift, L_c, equals $-2\pi\rho bUC(ik)Q$, where $C(ik)$ equals $K_1(ik)/(K_0(ik)+K_1(ik))$ and is known as Theodorsen's function.

The restriction $Re(s)>0$ in Eq. (3) implies that the assumed airfoil motion is strictly divergent. However, the application of the theory to simple harmonic motions [i.e., $s=ik(U/b)$] was accepted, since it agreed well with experimental flutter boundaries. The subsequent claim by Luke and Dengler[4] that the Theodorsen theory could be applied to the calculation of airloads due to stable airfoil motions [where $Re(s)<0$] could not be reconciled with the explicit airfoil motion assumed by Theodorsen, and their claim was rejected.

Earlier, Sears used the technique of Laplace transformation to obtain new derivations of indicial loading functions.[5] Sear's presentation is essentially a derivation of the generalized Theodorsen function. Making the change of variables $\xi=x_0-Ut$ in Eqs. (1) and (2) gives

$$Q(\tau)=-\frac{U}{2\pi b}\int_0^{\tau}\sqrt{\frac{(\tau-t)+\dfrac{2b}{U}}{\tau-t}}\,\gamma_w(t)\,dt \tag{4}$$

and

$$L_c(\tau)=\rho U^2\int_0^{\tau}\frac{(\tau-t)+\dfrac{b}{U}}{\sqrt{(\tau-t)^2+\dfrac{2b}{U}(\tau-t)}}\,\gamma_w(t)\,dt \tag{5}$$

where $\tau=(x_0-b)/U$. These expressions are convolution integrals, so the unknown transform $\ell[\gamma_w(t)]$ may be eliminated from the transforms of Eqs. (4) and (5). The transforms of the square root expressions are modified Bessel functions, and the circulatory lift is determined to be $L_c(s)=-2\pi\rho bUC(\bar{s})Q(s)$ where $\bar{s}=sb/U$. The function $C(\bar{s})$ is identical to the Theodorsen function evaluated for arbitrary values of s and is termed the generalized Theodorsen function. In evaluating the transforms of the functions in Eqs. (4) and (5), the restriction that $Re(s)>0$ is encountered. However, the modified Bessel functions are defined and analytic throughout the s-plane except for a branch cut along the negative real axis and, by analytic continuation, $C(\bar{s})$ is the unique transfer function relating $Q(s)$ to $L_c(s)$ for arbitrary values of s. It should be noted that no explicit form of airfoil motion is assumed as in Theodorsen's derivation and that no inconsistency results from the extension to convergent oscillations.

Inversion Integral for Unsteady Aerodynamic Loads

The Laplace inversion integral may be applied to calculate transient airloads

$$L_c(t)=\frac{1}{2\pi i}\int_{\sigma_1-i\infty}^{\sigma_1+i\infty}L_c(s)e^{st}\,ds \tag{6}$$

The contour integral may be evaluated by applying Cauchy's integral theorem to the contour shown in Fig. 1. The damped complex conjugate poles are representative of poles which may be introduced by $A(s)$. The contour passes above and below the branch cut along the negative real axis, making the integrand single valued within the contour. Examples of transient airloads due to stable oscillations are given in Ref. 1.

Root Loci of Aeroelastic Loads

The equations of motion of a three-degree-of-freedom section involving plunge, rotation, and trailing-edge flap deflection may be written as follows

$$[Ms^2+K-A(s)]X(s)=G\beta_c(s) \tag{7}$$

where $M,K,$ and G are 3×3 inertia, stiffness, and control distribution matrices, respectively, $A(s)$ is the matrix of aerodynamic load coefficients, $X^T(s)=[h(s)\alpha(s)\beta(s)]$, and $\beta_c(s)$ is a torque command input to the flap. The elements of $A(s)$ involve the nonrational function $C(\bar{s})$. Since $C(\bar{s})$ is valid for arbitrary values of s, the poles of the system may be determined by locating the zeroes of the determinant of the matrix of coefficients of Eq. (7). This was accomplished numerically with a gradient search algorithm, and Fig. 2 shows such a root locus vs $U/b\omega_\alpha$. As $U/b\omega_\alpha$ increases, all three modes initially move into the left half plane, with the plunge mode fluttering at $U/b\omega_\alpha\cong3.0$. Figure 2 indicates quantitatively the stability of the section at both subcritical and supercritical flutter conditions, since the method determines the true modes of the aeroelastic system. This is in contrast to methods that use simple harmonic aerodynamic loads, such as the U-g method, which yield quantitative information only at the flutter point.

The matrix of coefficients in Eq. (7) may be inverted and the equation solved for the transformed states, $X(s)$.

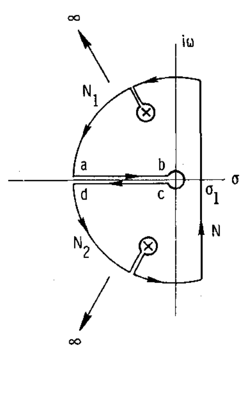

Fig. 1 Contour deformation used to evaluate inversion integral for incompressible flow.

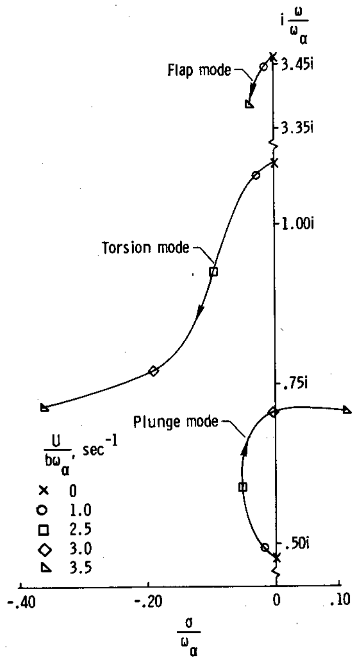

Fig. 2 Locus of roots of a three-degree-of-freedom section vs $U/b\omega_\alpha$ in incompressible flow. $\mu=40$, $a=-0.4$, $c=0.6$, $x_\alpha=0.2$, $r_\alpha^2=0.25$, $x_\beta=0.0125$ $r_\beta^2=0.00625$, $\omega_\alpha=100$ rad/sec, $\omega_h=50$ rad/sec, $\omega_\beta=300$ rad/sec.

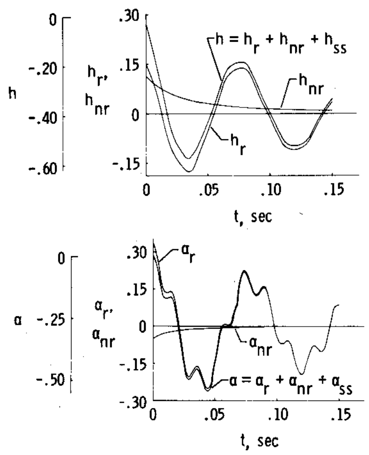

Fig. 3 Rational, nonrational, and total plunge and pitch due to a step command to the flap. $M=0$; $h_{ss}=-0.39$; $\alpha_{ss}=-0.28$.

Transient airfoil motions may then be calculated by evaluating the inversion integral along the deformed contour in Fig. 1. Figure 3 shows the calculated plunge and rotation response of the section of Fig. 2 due to a step input command to the flap. The responses are composed of rational portions (due to the poles contained within the contour) and nonrational portions (due to the integral along the branch cut). The nonrational portions do not participate in the oscillatory response characteristic of fluttering airfoils.

Generalized Compressible Unsteady Aerodynamics

It is natural to extend the technique of Laplace transformation of the study of loads due to arbitrary airfoil motions in compressible flow. The transformed three-dimensional linearized partial differential equation of unsteady aerodynamics is

$$\Phi_{xx}+\Phi_{yy}+\Phi_{zz}-(s^2/a^2)\Phi-(2Ms/a)\Phi_x-M^2\Phi_{xx}=f(x,y,z,0) \tag{8}$$

where $\Phi(x,y,z,s)=\ell[\varphi(x,y,z,t)$ and

$$f(x,y,z,0)=-(s/a^2)\varphi(x,y,z,0) - (1/a^2)\varphi_t(x,y,z,0)-(2M/a)\varphi_x(x,y,z,0) \tag{9}$$

The transformed boundary condition is

$$\ell[w_a(x,y,t)]=\Phi_z|_{z=0}=\left(s+U\frac{\partial}{\partial x}\right)\ell[z_a(x,y,t)]-z_a(x,y,0) \tag{10}$$

The terms $f(x,y,z,0)$ in Eq. (8) and $z_a(x,y,0)$ in Eq. (10) are initial conditions resulting from the Laplace integral transform. Since Eq. (8) is linear, the solution can be obtained as a superposition of solutions

$$\Phi(x,y,z,s)=\Phi_1(x,y,z,s)+\Phi_2(x,y,z,s) \tag{11}$$

where Φ_2 is regarded as a known function chosen to satisfy Eq. (8) subject to the boundary condition

$$\left[\frac{\partial}{\partial z}\Phi_2\right]_{z=0}=-z_a(x,y,0) \tag{12}$$

Then the solution Φ_1 satisfies a boundary-value problem which is formally identical to that resulting from the assumption of simple harmonic motion with the replacement of $i\omega$ by $s=\sigma+i\omega$. Thus, digital programs that calculate airloads due to simple harmonic motions may be modified in a fairly straightforward manner to calculate airloads due to arbitrary motions corresponding to the solution Φ_1.

Edwards[1] applies this technique to Garrick and Rubinow's[6] solution for two-dimensional supersonic flow. Airloads for complex values of $s=\sigma+i\omega$ and root loci of aeroelastic modes as a function of Mach number are presented in Ref. 1.

The loads resulting from the Φ_1 solution are linear with respect to the transformed airfoil motions, while the loads resulting from the Φ_2 solution are linear with respect to the initial conditions of the motion. Thus, the resulting airloads will take the form $A_1X(s)+A_2x(0)$ where $X(s)$ and $x(0)$ are $n\times1$ vectors of normal coordinates. As a consequence, only the airloads corresponding to the Φ_1 solution are required to determine stability, and computer programs modified as indicated may be used to calculate these loads. This provides a new and exact technique for the calculation of subcritical and supercritical flutter modes.

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Dynamic Characteristics of Rotor Blades: Integrating Matrix Method

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Nomenclature

EI = flapwise bending stiffness
e = distance between mass and elastic axis, positive when mass lies ahead of elastic center

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